Mass consumption and bounded learning by doing:

Some demand-side implications of income distribution for growth

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Abstract

Stylized facts relevant to the analysis of economic growth traditionally focus on the supply-side of the economy. Little reference is made to mass consumption which has accompanied both industrial revolutions, and which today is a key feature of economic development in large emerging countries. This paper provides an endogenous growth model where supply (structural change) and demand (middle class consumption) interact to bring out a multi-industry flying-wild-geese development pattern where each industry may learn from its own experience and/or from others. An industry learning curve is assumed to evolve over time with cumulative industry output which in turn depends on the income distribution. To that end, we relax the assumption of homothetic preferences that neutralizes demand in the long-run. We discuss the implications of income distribution on an industry learning curve in a set up where a society of mass consumption may arise as a consequence of horizontal demand complementarities and technological spillovers across industries. Eventually, the survival function of the income distribution determines each industry learning curve. Only a non-degenerate, i.e. neither perfectly equal nor completely unequal, distribution of income will yield long-run growth. There is an inverted-U relationship between inequality and growth. The rate of growth ultimately depending on the size of the middle class which creates the conditions for mass consumption, scale economies and learning by doing as sources of sustained growth.

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1 Introduction

On the one hand, the emergence of a middle class has characterized the industrial revolutions experienced by the Western economies during the Eighteenth and Nineteenth centuries. On the other hand, the beginning of the twenty-first century marks the end of two centuries of hegemony of these Western economies. A decentering of the world has already started that disrupts the economic world balances. "The current economic takeoffs in China and India began with about 1 billion people in each country and doubled output per capita in less than 20 years - an economic force affecting a much larger population than the Industrial revolution did... Increasingly, the most important engine of growth for countries of the South is their domestic market. The middle class is growing in size and median income. By 2025, annual consumption in emerging markets is estimated to rise to $30 trillion. By then, the South will account for three-fifths of the 1 billion households earning more than $20000 a year." (Human Development Report 2013). Nowadays, it is common in the press as well as in policy circles to wonder about the situation of the middle class and its role to play in the economic development. However, the concept of middle class has not received yet the attention it deserves in our thinking about economic growth.

Why? Because the study of economic growth traditionally comes in a long-run perspective. Thus the supply side has been given the priority over the demand side, and, to put it mildly, demand has so far been considered as unimportant for economic growth. In their presentation of the "new Kaldor facts", Jones and Romer (2010) emphasize the role of ideas rather than objects as well as the importance of institutions and human capital. However, what about mass consumption? The supply-side view is not designed to highlight the mechanisms induced by mass consumerism on economic growth. Hence, this issue is nearly absent from both neoclassical and new (endogenous) theories of growth[1].

A proposed runway to investigate such interactions is to relax the traditional assumption of homothetic preferences which neutralizes demand on long-run growth; that is, to take into account the impact of household income on the composition of his consumption basket, and consequently, the role of the size of the market on those industries in which an economy will tend to specialize. This orientation builds on the work of Kevin Murphy, Andrei Shleifer and Robert Vishny (1989a, henceforth MSV, and 1989b) who have formalized works on demand linkages of early development economists such as Paul Rosenstein-Rodan (1943), Albert Hirschman (1958) whose theories stress scale economies and complementarities as deep causes of economic growth[2], and Walt Rostow (1960) for whom a society of mass consumption must be the final stage of eco-

[1] In his introduction of the Handbook of Economic Growth (2005), Robert Solow regrets the lack of interest of the profession for interactions between supply and demand in the medium run.

[2] As argued by Paul Krugman in his Ohlin lectures on the fall and rise of development economics "[...] The reason that the development theory that emerged in the 1940s... failed to 'make it' into mainstream economics was the inability of their creators to express their ideas in a way suitable for the modeling techniques available at that time..." Paul Krugman 1995, p.6, in *Development, geography, and economic theory.*

nomic development of a nation.

It was not until recently that models of growth have relaxed the assumption of homothetic preferences in models of economic growth and structural change. The adoption of a hierarchic structure of preferences where poor consumers devote most of their expenditures toward low income elasticity goods and its impact on economic growth is examined, for instance, in Zweimüller (2000) and Foellmi and Zweimüller (2006 and 2008). We follow in their footsteps. Our setup allows for hierarchies of needs in consumption and economic growth is an endogenous outcome of the economic system. It differs from them though in two ways. First, technical progress is not driven by innovations. There is no R&D sector as in Romer (1990) that generates blue prints for new inputs ('new methods to satisfy wants' in Zweimüller 2000) as a result of voluntary profit-motivated innovations. Instead, technical progress is a by-product of the economic activity where learning at the industry level is the source of productivity gains. (See, for example, Dutton and Thomas 1984, Bahk and Gort 1993, and Thompson 2001 who provide evidence for a significant impact of learning by doing on productivity.) More specifically, learning is assumed to be bounded at the industry level and to take place only in those industries where demand is high enough so that a firm becomes able to take advantage of internal economies of scale, i.e. to lower unit costs of production, thanks to mass consumption[3]. Thus how much an industry can gain in terms of cost reduction is due to increased knowledge resulting from cumulative output which is driven by the size of the market that in turn depends on the income distribution. In fact, the survival function of the income distribution determines each industry learning curve. Secondly, our modelling involves no saving/investment. Therefore, we refer here to an experience curve brought about by labor learning which is closely intertwined with scale economies in that scale, i.e. the market size, also contributes to learning. It thus differs from the concept of learning-by-doing hypothesized by Arrow (1962) who considers cumulative investment as an alternative to cumulative output and, therefore, from Paul Romer's seminal paper (1986) which pioneered endogeneous growth theory.

The present paper builds on the static frameworks of MSV and Desdoigts and Jaramillo (2009). Of the background literature, Lucas (1993) and Matsuyama (2002) are the papers most closely related to ours. First, we follow MSV static setup by introducing pecuniary externalities working via the buying power of a middle class to eventually determine the extent of horizontal complementarity across all industries of the economy (see also the insightful paper by Matsuyama 1995). Secondly, this kind of argument which captures how one thing leads to another is central to Matsuyama (2002) who shows what characteristics of the distribution of income can lead to the emergence of an economy

[3] It should be noted that the classic case study chosen by Thompson (2001), relies on an extremely well-documented program, namely the Liberty shipbuilding program of World War II, which has been characterized by a dramatic and sudden output expansion particularly because of growth of primary demand. Without such a strong expansion of demand, this now-famous case study may not have become such an interesting natural experiment in terms of measuring the contribution of learning to increases in labor productivity.

of mass consumption. This economic development requires gains in productivity through learning which, by lowering prices, gives access to a consumption basket consisting of different goods depending on the household income, and not necessarily to the consumption of a greater quantity of the same goods. With hierarchical preferences which rank goods in order of priority, a greater variety of goods becomes available to households, and the income effect of lower prices leads new industries to develop.

On the one hand, results obtained by Thornton and Thompson (2001) in Wartime shipbuilding show that learning spillovers are a significant potential source of productivity growth. On the other hand, recent empirical evidence by Wolff (2011) suggests a strengthening of technological spillover effects in the US economy over the period 1958-2007. One characteristic of our model is that, similarly to Stokey (1988) and Lucas (1993) among others, there are technological spillovers across sectors that are ruled out in Matsuyama (2002). Our development process follows however a pattern fairly similar to Matsuyama (2002) of a flying-wild-geese where the increasing returns technology is implemented in industries one after the other. The flying geese model of economic development was first coined by Kaname Akamatsu in the 1930s, and gained popularity in the 1960s. It was initially based on Japan's experience in the development of its woolen industry which has then be applied to the sequential appearance and development of industries leading to the interindustry aspect of the flying geese model with demand linkages and complementarities of different products being the driving forces of economic development[4].

Thus we assume that learning is bounded at the industry level but suppose there are learning spillovers. It is appropriate to put our model in correspondence with the bounded learning model by Alwyn Young (1991), where all goods are not produced on a given date because they would be too expensive to produce. As long as knowledge accumulates in the different industries, the increase of knowledge reduces the labour unit cost of all goods including those goods whose cost was previously prohibitive. There comes a time when these goods can be produced at reasonable cost. In our model, all goods may be available as soon as there exists a consumer for it. It is only in those sectors where demand is strong enough to cover the fixed costs involved in the implementation of the increasing returns technology that the learning takes place. Therefore, it is a consequence of the substantial economic activity caused by mass consumption in these industries. The implementation of increasing returns technologies that are substituted for constant returns technologies is used here as a metaphor for structural change. The Indian pharmaceutical industry, in particular in the field of generic drugs, or its telecommunications industry provide examples of the importance of the size of the internal market in the area of apprenticeship in emerging markets, which is itself a source of increased knowledge-based productivity.

[4] Combining the interindustry flying geese patterns of industrialization in economically interdependent countries results in the international aspect, that is to say the catching up of developing countries with the West (Schröppel and Mariko 2002). Still, note that there is no international linkage in the present model which remains a closed-economy growth model.

However, do more equal distributions generate more demand throughout the economy, which in turn, leads to learning across a wider range of industries, i.e. higher productivity, and in fine faster growth? We show in this paper why this may not be necessarily true. Only a non-degenerate, i.e. neither perfectly equal nor completely unequal, distribution of income will yield long-run growth. Moreover, there is an inverted-U relationship between inequality and growth[5]. The rate of growth ultimately depending on the size of the middle class which creates the conditions for mass consumption, scale economies and learning by doing as sources of sustained growth.

This paper thus contributes to the theoretical debate about the impact of inequality on economic growth which has developed over the last two decades[6]. It is organized as follows. In Section 2, we present our framework in which these issues can be formally analyzed. Section 3 discusses our set up of knowledge productivity growth. Section 4 characterizes the relationships between mass consumption, experience curves at the industry level, learning spillovers, and the steady-state aggregate rate of growth. The last section is devoted to analyzing the conditions with regard to technological progress, which underlie sustainable economic development when mass consumption and not population growth harms the environment.

2 The model

2.1 Households' non-homothetic preferences, wealth, and budget constraint

Our framework assumes that all households have the same preferences. The preference side is modeled via a utility function which is defined over a continuum of indivisible goods $q \in (0, \infty)$ such that, at each date t,

$$V_t = \int_0^\infty \frac{1}{q} x_t^q dq, \tag{1}$$

where x_t^q as an indicator function which takes in values of either one or zero according to:

$$x_t^q = \begin{cases} 1 \text{ if the agent consumes } q \\ 0 \text{ otherwise} \end{cases} .$$

Thus a household's utility increases with the range of goods $(0, q)$ it consumes and not with the consumption of a single good q. Consumption is hierarchically structured; that is, needs are ordered so that the proportion of income that

[5] Empirical studies by Barro (2000), Banerjee and Duflo (2003), and Voitchovsky (2005), amongst others, find a non-linear relationship between inequality and growth. Interestingly enough, Voitchovsky investigates the importance of the shape of the income distribution over a single inequality statistic of inequality on economic growth.

[6] See, for example, the surveys by Aghion *et al.* 1999, and, more recently, Voitchovsky 2009.

households spend on lower-indexed goods or, equivalently, on goods with lower income elasticities of demand, decreases with a household's income. Different goods have different priorities in consumption and richer households can consume more than the bundle of goods available to poorer households (Bertola, Foellmi, and Zweimüller 2006, Chapter 12).

Human capital is the only input, and the economy is endowed with an amount $\overline{h}L$, where L is the entire population and $\overline{h}$ is the average level of human capital in the economy, that we normalize to one. Moreover, h_γ denotes a household's human capital endowment which is assumed to be constant over time. It is given by

$$h_\gamma = \gamma \overline{h} L = \gamma L,$$

where γ is the share of human capital of a type-γ household, and $\underline{\gamma} \leq \gamma < \infty$. The total stock of human capital in the economy is distributed according to the cumulative distribution function $G(\gamma)$ which is assumed to be exogenous and constant over time. Therefore, each household is identified by its type γ. At each date t, the labor income of a type-γ household is given by:

$$w_t h_\gamma = w_t \gamma L,$$

where w_t is the wage per unit of human capital.

The nominal income of a type-γ household is defined as

$$Y_t^\gamma = \gamma(w_t L + \Pi_t),$$

where Π_t is the aggregate amount of profits realized by all firms from all industries q in the economy. Profits are redistributed to households up to their type γ.

Define $(0, q_t^\gamma)$ as the set of goods purchased by a type-γ household. The budget constraint which describes the consumption options available to this household with income Y_t^γ can be written as:

$$\int_0^{q_t^\gamma} p_t^q x_t^q dq = \gamma(w_t L + \Pi_t). \tag{2}$$

2.2 Factor supply, technology, market structure, and the equilibrium price

In this section, we adopt the now standard production technologies and market structure proposed by Murphy, Shleifer, and Vishny (1989b) in their formalization of the big push as well as in their aforementioned more closely related paper to ours. We assume that each good q can be produced with two production functions. The former exhibits constant returns to scale (CRS). One unit of good q requires α/A_t units of human capital, with $\alpha > 1$ and A_t is knowledge-based productivity at time t. The alternative production technology exhibits increasing returns to scale (IRS). Formally, $1/A_t$ units of human capital are required to produce one unit of good q. Nevertheless, in order to produce at such a marginal cost, a firm must also be able to cover a fixed cost equal to F/A_t units of human capital.

On the one hand, each good q may be produced by a competitive fringe of firms with the CRS technology. Then, the free-entry equilibrium number of firms satisfies the zero-profit condition, and the equilibrium price is equal to the average cost; that is,

$$p_t^q = p_t = \alpha w_t / A_t. \tag{3}$$

On the other hand, we show that if the distribution function $G(\gamma)$ is smooth enough which rules out perfect equality, there is a unique Nash equilibrium for a monopoly implementing the IRS technology, which consists in setting the price at the same level of the competitive fringe (see Appendix 1).

2.3 Market demand and the static output multiplier

Whenever the demand is high enough to cover the fixed cost, a good q will be produced by a monopolist which will implement the IRS technology. Note that if $L > F/(\alpha - 1)$, a good q is produced using the IRS technology if and only if the demand for this good at time t, denoted X_t^q, is such that the following minimum efficient scale is satisfied:

$$\frac{(\alpha - 1)w_t}{A_t} X_t^q - \frac{F w_t}{A_t} \geq 0 \Leftrightarrow X_t^q \geq \frac{F}{\alpha - 1}. \tag{4}$$

At each time t, there is a marginal good q_t^* such that the break-even condition $X^{q_t^*} = F/(\alpha - 1)$ holds true. Within our dynamic model, such an infant industry can be said to have attained the take-off stage (see below). Note that $X^{q_t^*}$ is exogenous and constant over time. Then, industries which produce goods $q \leq q_t^*$, respectively $q > q_t^*$, use the IRS, respectively the CRS, production technology. Following MSV, we define γ_t^* as the share of income held by this marginal household whose purchasing power allows it to exactly purchase the range of goods $(0, q_t^*)$, where

$$q_t^* = \frac{w_t \gamma_t^* (L + \Pi_t / w_t)}{p_t} = \frac{A_t}{\alpha} \gamma_t^* \left(L + \frac{\Pi_t}{w_t} \right), \text{ and } \frac{w_t}{p_t} = \frac{A_t}{\alpha}. \tag{5}$$

We also define the upper class to be the set of households of type greater than γ_t^*. There is an amount N_t^* of such households, where

$$N_t^* = (1 - G(\gamma_t^*))L. \tag{6}$$

Their purchasing power allows them to buy goods produced with the IRS technology as well as goods with higher income elasticity of demand which are produced using the CRS technology. We therefore have the following break-even condition which is time-independent. We thus get rid of the t notation in both variables N^* and γ^*.

$$X^{q_t^*} = N^* = (1 - G(\gamma^*))L = \frac{F}{\alpha - 1}. \tag{7}$$

New models of economic growth along the lines of Romer (1990) emphasize the increase in available varieties of goods as a metaphor of economic growth. Overall, what matters in the new theories of growth is the nature of imperfect competition. Monopolistic competition with its zero-profit condition in equilibrium prevails extensively in new growth theories. Into our model, the variable of interest is not the number of varieties produced in equilibrium, but the number of goods produced with the IRS technology and, as a result, the equilibrium profits generated by industries which are able to implement the IRS technology allowing firms to achieve internal to the firm economies of scale.

Aggregate profits in the economy are the sum of profits realized by those industries which produce goods q in the range $(0, q_t^*)$:

$$\begin{aligned}\Pi_t &= p_t \int_0^{q_t^*} X_t^q dq - \int_0^{q_t^*} \frac{w_t}{A_t}(X_t^q - F)dq \\ &= (\alpha - 1)\frac{w_t}{A_t}\int_0^{q_t^*} X_t^q dq - \frac{w_t}{A_t}\int_0^{q_t^*} F dq,\end{aligned}$$

where the demand for each good q_t at time t is given by

$$X_t^q = (1 - G(\gamma_t^q))L, \tag{8}$$

and where $\gamma_t^q = p_t q_t/(w_t L + \Pi_t)$ is the share of income of the poorest household whose purchasing power is high enough to exactly purchase $(0, q_t)$.

Now combining the above profit expression with (5) and (6) yields

$$\begin{aligned}\frac{\Pi_t}{p_t} &= \frac{A_t}{\alpha w_t}\left((\alpha - 1)\frac{w_t}{A_t}\int_0^{q_t^*} X^{q_t} dq - \frac{w_t}{A_t}\int_0^{q_t^*} F dq\right) \\ &= \frac{\alpha - 1}{\alpha}\int_0^{q_t^*} X^{q_t} dq - \frac{1}{\alpha}\int_0^{q_t^*} F dq \\ &= \frac{\alpha - 1}{\alpha}\frac{(w_t L + \Pi_t)}{p_t} T,\end{aligned} \tag{9}$$

where $T = L\int_{\underline{\gamma}}^{\gamma^*} \gamma dG(\gamma)$ is defined as the share of income held by those households of type smaller than γ^* whose income is entirely devoted to purchase goods of mass consumption, i.e. in the range $(0, q_t^*)$. From this definition, we deduce:

$$\frac{\Pi_t}{p_t} = \frac{\alpha - 1}{\alpha}\frac{1}{(1 - \frac{\alpha - 1}{\alpha}T)}\frac{w_t L}{p_t},$$

where the multiplier is defined by

$$M = 1/(1 - \frac{\alpha - 1}{\alpha}T),$$

which is independent of time.

The average real income per capita (y_t) of the economy is therefore proportional to the multiplier and the knowledge-based productivity at time t. It takes the form

$$y_t = \frac{Y_t}{p_t L} = \frac{w_t L + \Pi_t}{p_t L} = \frac{1}{1 - \frac{\alpha - 1}{\alpha} T} \frac{A_t}{\alpha}. \tag{10}$$

The higher T, the higher is y_t. In the next section, we specify the rate of growth of A_t so that we become able to study the implications of inequality for growth and patterns of industrialization.

2.4 Experience at the industry level and cumulative output

It is assumed that learning occurs only in those industries which produce goods that are generated from the IRS technology. At each date t, such learning leads to an accumulation of experience at the industry level denoted by E_t^q to be discussed below. We also assume that this accumulated experience diffuses instantaneously to all other firms, i.e. there are intersectoral spillovers. Accordingly, the level of knowledge-based productivity in the economy A_t is the same in all industries and is equal to the sum of experiences gathered over time by all industries producing goods q and having implemented the IRS technology. Thus learning acts as a productivity-enhancing factor in the above production functions and firms gain from the effects of experience via lower costs. More specifically, the accumulated stock of knowledge at time t is defined by:

$$A_t = \int_0^{q_t^*} E_t^q dq. \tag{11}$$

We follow the literature on learning to describe progress at the industry level. Experience increases with cumulative production levels[7]. We adopt the following functional form for the experience accumulated in industry q over a time interval (t_q^*, t) (see Thompson 2010):

$$E_t^q = \frac{\varepsilon + \lambda \widetilde{E} \displaystyle\int_{t_q^*}^{t} X_v^q dv}{1 + \lambda \displaystyle\int_{t_q^*}^{t} X_v^q dv}, \tag{12}$$

where $\lambda > 0$ describes the learning rate, t_q^* denotes the date at which the IRS technology has been adopted for the first time by the industry which produces good q, and X_v^q is the level of output produced in this industry at time v. First, note that $\int_{t_q^*}^{t} X_v^q dv = 0$ implies $E_t^q = \varepsilon$. Secondly, E_t^q is monotonically increasing

[7] Note that in our theoretical framework, experience gained prior to some time t may equivalently be expressed in terms of cumulative units of human capital.

and concave with an asymptote[8] when $\int_{t_q^*}^t X_v^q dv \to \infty$ which is equal to $\widetilde{E}$. Hence, learning in every industry is bounded, the upper bound being defined by $\widetilde{E}$ which is assumed to be exogenous and constant both across industries and over time. However, recall that it is not bounded at the aggregate level as the learning which occurs in one industry spills over across industries.

First, let us assume $\varepsilon = 0$, i.e. no learning occurred before a good q has been produced with the IRS technology. The experience curve at the industry level is described by[9]:

$$E_t^q = \frac{\lambda \widetilde{E} \int_{t_q^*}^t X_v^q dv}{1 + \lambda \int_{t_q^*}^t X_v^q dv} \Rightarrow \frac{\dot{E}_t^q}{E_t^q} = \frac{X_t^q}{\left(1 + \lambda \int_{t_q^*}^t X_v^q dv\right) \int_{t_q^*}^t X_v^q dv}.$$

Secondly, and for ease of use, we adopt a linear approximation to (12) near $\int_{t_q^*}^t X_v^q dv = 0$[10]. More specifically, we define the experience accumulated in

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[8]Let us denote $\int_{t_q^*}^t X_v^q dv$ by χ_t^q, it is easily checked that

$$\frac{\partial E_t^q}{\partial \chi_t^q} > 0 \text{ and } \frac{\partial^2 E_t^q}{\partial \left(\chi_t^q\right)^2} < 0.$$

Using l'Hôpital's rule, we have:

$$\lim_{\chi_t^q \to \infty} E_t^q = \lim_{\chi_t^q \to \infty} \frac{\partial(\varepsilon + \lambda \widetilde{E} \chi_t^q)/\partial \chi_t^q}{\partial(1 + \lambda \chi_t^q)/\partial \chi_t^q} = \widetilde{E}.$$

[9]We have:

$$\dot{E}_t^q = \frac{\lambda \widetilde{E} X_t^q (1 + \lambda \int_{t_q^*}^t X_v^q dv) - \lambda X_t^q (\lambda \widetilde{E} \int_{t_q^*}^t X_v^q dv)}{(1 + \lambda \int_{t_q^*}^t X_v^q dv)^2} = \frac{\lambda \widetilde{E} X_t^q}{(1 + \lambda \int_{t_q^*}^t X_v^q dv)^2}$$

[10]Let us rewrite (12) as a function of χ_t^q and assume $\varepsilon = 0$, such that

$$E_t^q = \frac{\lambda \widetilde{E} \chi_t^q}{1 + \lambda \chi_t^q} \text{ and } \frac{\partial E_t^q}{\partial \chi_t^q} = \frac{\lambda \widetilde{E}}{1 + \lambda \chi_t^q}.$$

Hence, the linear approximation to E_t^q near $\chi_t^q = \overline{\chi}_t^q$ is given by

$$E_t^q \simeq \frac{\lambda \widetilde{E} \overline{\chi}_t^q}{1 + \lambda \overline{\chi}_t^q} + \frac{\lambda \widetilde{E}}{1 + \lambda \overline{\chi}_t^q} (\chi_t^q - \overline{\chi}_t^q).$$

Thus, for $\overline{\chi}_t^q = 0$, we have

$$E_t^q \simeq \lambda \widetilde{E} \chi_t^q,$$

whereas if $\overline{\chi}_t^q \to \infty$ we obtain

$$E_t^q \simeq \widetilde{E}.$$

industry q at time t based on the cumulative level of production since it has adopted the IRS technology. This yields

$$E_t^q = \begin{cases} \lambda \widetilde{E} \displaystyle\int_{t_q^*}^t X_v^q dv \text{ if } \displaystyle\int_{t_q^*}^t X_v^q dv < 1/\lambda \\ \widetilde{E} \text{ otherwise} \end{cases}. \tag{13}$$

At each time t, among those industries which use the IRS technology, we can distinguish between two groups of industries. The former group includes those industries where learning is exhausted and has reached the upper bound, whereas, in the latter group, we find these industries where the accumulated experience has not reached yet the upper bound. This group of industries lies within the range $(\widetilde{q}_t, q_t^*)$, where $\widetilde{q}_t$ is defined by

$$E_t^{\widetilde{q}} = \widetilde{E} \Leftrightarrow \int_{t_{\widetilde{q}}^*}^t X_v^{\widetilde{q}} dv = 1/\lambda, \tag{14}$$

and where $t - t_{\widetilde{q}}^*$ denotes the time elapsed since good $\widetilde{q}_t$ has been first produced using the IRS technology. In words, as soon as $\int_{t_q^*}^t X_v^q dv \geq 1/\lambda$, learning in industry q has reached the upper bound $\widetilde{E}$.

In addition to q_t^*, there exists another marginal industry $\widetilde{q}_t$ which is defined as the most recent industry at time t where learning has declined to zero. Solving (14) yields a solution for $\widetilde{q}_t$. Similarly to the type-γ^* household, there is therefore another key marginal household of type $\widetilde{\gamma}_t$, with $\underline{\gamma} \leq \widetilde{\gamma}_t \leq \gamma^*$, whose purchasing power allows him to buy exactly the range of goods $(0, \widetilde{q}_t)$. If households of type γ^* and above constitute the upper income class, mass consumers may now be divided into a low income class which includes households of type between $\underline{\gamma}$ and $\widetilde{\gamma}_t$, and a middle income class where we find households of type ranking from $\widetilde{\gamma}_t$ to γ^*.

3 Knowledge-based productivity growth

Knowledge-based productivity A_t is a function of the experience accumulated by all industries in which learning occurs and already took place at time t. We thus rewrite (11) as

$$A_t = \int_0^{q_t^*} E_t^q dq = \widetilde{E}\widetilde{q}_t + \int_{\widetilde{q}_t}^{q_t^*} E_t^q dq.$$

Changes in A_t are the result of the experience accumulated in the economy at time t; that is,

$$\begin{aligned} \dot{A}_t &= \widetilde{E}\dot{\widetilde{q}}_t + \int_{\widetilde{q}_t t}^{q_t^*} \dot{E}_t^q dq + E_t^{q_t^*} \dot{q}^* - E_t^{\widetilde{q}_t} \dot{\widetilde{q}}_t \\ &= \int_{\widetilde{q}_t}^{q_t^*} \dot{E}_t^q dq = \lambda \widetilde{E} \int_{\widetilde{q}_t}^{q_t^*} X_t^q dq, \end{aligned}$$

with

$$\dot{E}_t^q = \begin{cases} \lambda \widetilde{E} X_t^q \text{ if } E_t^q < \widetilde{E} \text{ and } \widetilde{q}_t < q \leq q_t^* \\ 0 \text{ otherwise} \end{cases} \Leftrightarrow$$

$$\frac{\dot{E}_t^q}{E_t^q} = \begin{cases} X_t^q / \int_{t_q^*}^t X_v^q dv \text{ if } E_t^q < \widetilde{E} \text{ and } \widetilde{q}_t < q \leq q_t^* \\ 0 \text{ otherwise} \end{cases}.$$

The accumulated experience is a by-product of the economic activity in those industries where there is mass consumption. Given that $X_t^q = (1 - G(\gamma_t^q))L$, the evolution of demand addressed to industry q is at the origin of its learning curve which explicitly depends on the survival function of the distribution of income. This yields the following productivity rate of growth that arises from economywide learning by doing:

$$g_t = \frac{\dot{A}_t}{A_t} = \lambda \widetilde{E} \int_{\widetilde{q}_t}^{q_t^*} \frac{X_t^q}{A_t} dq = \lambda \widetilde{E} \int_{\widetilde{q}_t}^{q_t^*} \frac{(1 - G(\gamma_t^q))L}{A_t} dq. \tag{15}$$

At each time t, the rate of growth of knowledge-based productivity equals the amount of human capital (excluding the fixed costs) required to produce the quantity X_t^q in industries using the IRS technology and for which learning takes place. Thus unit cost reductions due to increased knowledge result from increased cumulative output brought about by both learning and economies of scale.

Whereas q_t^* evolves with time, recall that γ^* is constant over time and equal to $q_t^*/y_t L$ (see (2) and (10)). We use the change of variables $\gamma = q_t^\gamma / y_t L$ to rewrite (15):

$$g_t = \lambda \widetilde{E} \frac{y_t L}{A_t} \int_{\widetilde{\gamma}_t}^{\gamma^*} (1 - G(\gamma)) L d\gamma. \tag{16}$$

An integration by parts shows that:

$$\begin{aligned} \int_{\widetilde{\gamma}}^{\gamma^*} (1 - G(\gamma)) L d\gamma &= \left(\gamma (1 - G(\gamma))|_{\widetilde{\gamma}_t}^{\gamma^*} + \int_{\widetilde{\gamma}_t}^{\gamma^*} \gamma g(\gamma) d\gamma \right) L \\ &= \gamma^* (1 - G(\gamma^*))L - \widetilde{\gamma}(1 - G(\widetilde{\gamma}_t))L + T - \widetilde{T}_t, \end{aligned}$$

where $\widetilde{T}_t = \int_{\underline{\gamma}}^{\widetilde{\gamma}_t} \gamma g(\gamma) L d\gamma$.

The growth rate of A_t thus becomes:

$$g_t = \lambda \widetilde{E} L \frac{M}{\alpha} \left[\gamma^* N^* + T - \left(\widetilde{\gamma}_t \widetilde{N}_t + \widetilde{T}_t \right) \right], \tag{17}$$

where $M/\alpha = y_t/A_t$. Note that $\widetilde{N}_t$ cannot exceed L which would imply $\widetilde{T}_t < 0$. Therefore, there is a maximum for g_t which is equal to:

$$g_{\max} = \lambda \widetilde{E} L \frac{M}{\alpha} \left[\gamma^* N^* + T - \underline{\gamma} L \right]. \tag{18}$$

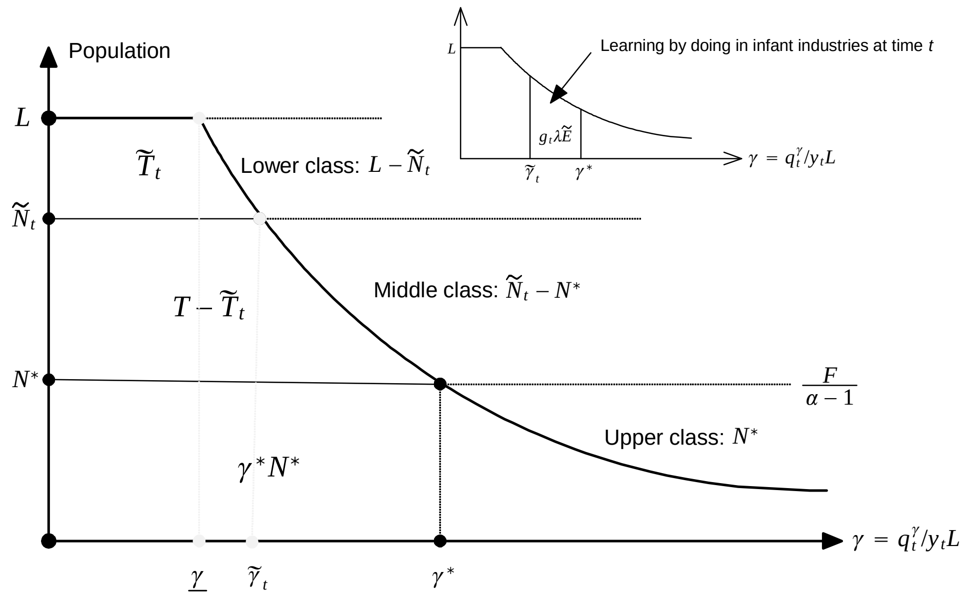


Figure 1: Implications of bounded learning and inequality for growth and structural change.

Similarly to the static model of MSV without learning, $\gamma^* N^* + T$ is the proportion of total income being spent in those industries having implemented the IRS technology. This income share is time-independent. The rate of growth at time t, therefore, depends positively on both the output multiplier and the proportion of income which is spent in industries where learning is not yet exhausted. Put differently, (17) provides us with a relationship between growth and mass consumption, where growth depends on the share of aggregate real income held by those households of type between $\widetilde{\gamma}_t$ and γ^*. Figure 1 summarizes where expenditures of various consumers go and also depicts those industries where learning takes place. Indeed, as alluded earlier, we have $\gamma = q_t^\gamma / y_t L$.

4 Mass consumption and sustained growth in the long run

4.1 The steady-state rate of growth

In the long run, as long as there is positive growth, an industry q goes through several stages. It begins with producing the good q with the CRS technology. At this stage, q can be considered as a luxury good and does not represent an essential need to consumers. It is produced for specific consumers, namely the highest income consumers N^*. Then, it goes through that stage where it becomes at some point in time the marginal industry q_t^*, i.e. the highest indexed industry at time t using the IRS technology. It produces q in an amount $X_t^{q^*}$ From then on, it starts to learn and to accumulate experience. Its apprenticeship will continue until exhaustion. That is, until it will have reached the upper bound $\widetilde{E}$. At the time t, when the industry just exhausted all its potential for learning, it becomes the marginal industry $\widetilde{q}_t$ whose good is purchased in quantity $X_t^{\widetilde{q}} = (1 - G(\widetilde{\gamma}_t))L$. What time does it take for an industry to reach the upper bound? The analysis below can be seen as a survival analysis where we are interested in the length of time before the upper bound $\widetilde{E}$ is reached by an industry once it has implemented the IRS technology.

The experience accumulated by one industry depends on its market size and growth rates over times prior to that time when it reaches $\widetilde{E}$. Recall from (13) that for all $q \in (\widetilde{q}_t, q_t^*)$, the experience accumulated at time t by the industry indexed by q is described by using the survival function $1 - G(\gamma_t^q)$:

$$\frac{E_t^q}{\widetilde{E}} = \lambda \int_{t_q^*}^{t} X_v^q dv = \lambda \int_{t_q^*}^{t} (1 - G(\gamma_v^q))Ldv.$$

We are thus interested in the learning duration, i.e. the length of the time period in-between t_q^* and t, where t must be such that

$$\frac{E_t^{\widetilde{q}}}{\widetilde{E}} = 1 \Leftrightarrow \int_{t_{\widetilde{q}}^*}^{t} X_v^{\widetilde{q}} dv = \int_{t_{\widetilde{q}}^*}^{t} (1 - G(\gamma_v^{\widetilde{q}}))Ldv = \frac{1}{\lambda}. \tag{19}$$

The demand in industry q at time t, with $\widetilde{q}_t \leq q \leq q_t^*$, depends on the law of motion of demand addressed to the industry q over the time interval (t_q^*, t). Let us rewrite the demand for that good at time t as a function of the average rate of growth between t_q^* and t of demand growth rates at each time v $(\dot{X}_v^q / X_v^q)$, where $t_q^* \leq v \leq t$:

$$X_t^q = X_{t_q^*}^q \exp(g_{t_q^*,t}^q (t - t_q^*)),$$

where $X_{t_q^*}^q = N^*$ and $g_{t_q^*,t}^q = (t - t_q^*)^{-1} \int_{t_q^*}^{t} (\dot{X}_v^q / X_v^q) dv$. Using the definition of $X_v^q = 1 - G(\gamma_v^q)$ and the change of variables $\gamma_v^q = q/y_v L$, the rate of growth of demand for a good $q > \underline{q}_v$ is given by[11]:

$$\frac{\dot{X}_v^q}{X_v^q} = \gamma_v^q \frac{g(\gamma_v^q)}{1 - G(\gamma_v^q)} \frac{\dot{y}_v}{y_v}, \tag{20}$$

[11] Let us us start with:

$$\dot{X}_v^q = \frac{\partial (1 - G(\gamma_v^{\widetilde{q}}))L}{\partial v} = -g(\gamma_v^{\widetilde{q}}) L \dot{\gamma}_v^{\widetilde{q}},$$

where $\gamma_v^q = q/y_t L$ implies $\dot{\gamma}_v^q = -(q/y_v^2 L)\dot{y}_v$. We get

$$\dot{X}_v^q = g(\gamma_v^{\widetilde{q}}) \left(\frac{q}{(y_v)} \frac{\dot{y}_v}{y_v} \right)$$

$$\Rightarrow \dot{X}_t^q = g(\gamma_v^{\widetilde{q}}) \gamma_v^q L \frac{\dot{y}_v}{y_v}, \text{ with } \gamma_v^q = q/y_t L.$$

where $X_v^q = (1 - G(\gamma_v^q))\, L$ and $g(\gamma_v^q)/(1 - G(\gamma_v^q))$ also known as the hazard rate is the number of households of type γ_v^q relative to the number of households whose income is greater than γ_v^q. In other words, if y_t increases by 1%, the demand for good q increases by γ_v^q times the hazard rate. The income elasticity of aggregate demand for a good q is either zero for industries indexed by $q \leq \underline{q}_v$ or equal to $\gamma_v^q g(\gamma_v^q)/\,(1 - G(\gamma_v^q))$ for all $q > \underline{q}_v$. We face here a technical difficulty, the income elasticity of demand for a good $q > \underline{q}_v$ evolves over time and depends on $G(\gamma_v^q)$.

At this stage, we choose to specify the distribution of income $G(\gamma)$ and adopt the Pareto distribution which exhibits useful properties as a functional form for income distribution. We specify $G(\gamma) = 1 - (\underline{\gamma}/\gamma)^\beta$ with $\beta > 1$, the shape parameter, and $\gamma \geq \underline{\gamma} > 0$, the scale parameter. The larger the value of parameter β, the more equal the distribution of income. Put differently, dispersion increases monotically as β decreases. Note that $\beta = \gamma_v^q g(\gamma_v^q)/\,(1 - G(\gamma_v^q))$, this implies:

$$\frac{\dot{X}_v^q}{X_v^q} = \beta \frac{\dot{y}_v}{y_v},$$

which thus becomes constant across all industries $q > \underline{q}_v$[12], and the income elasticity of demand addressed to each industry $q \in (\underline{q}_v, q_v^*)$ is equal to the Pareto shape parameter.

In the steady state equilibrium, we normally have

$$g = \frac{\dot{A}_t}{A_t} = \frac{\dot{y}_t}{y_t} = \frac{\dot{q}_t^*}{q_t^*} = \text{constant},$$

and, therefore, the differential equation (20) verifies in the Pareto case and in the steady state:

$$\begin{aligned} X_t^q &= N^* \exp(\beta g(t - t_q^*)) \\ &\Rightarrow \; X_t^{\widetilde{q}} = \widetilde{N}_t = N^* \exp(\beta g(t - t_{\widetilde{q}}^*)). \end{aligned} \tag{21}$$

As long as the income elasticity of demand for a good $q > \underline{q}_t$ is constant both over the time interval $(t_{\widetilde{q}}^*, t)$ and across industries, we are able to use (21)

This yields:

$$\frac{\dot{X}_t^q}{X_v^q} = \gamma_v^q \frac{g(\gamma_v^{\widetilde{q}})}{1 - G(\gamma_v^q)} \frac{\dot{y}_v}{y_v}.$$

[12] Another interesting property of the Pareto distribution in our framework is that:

$$\frac{\beta - 1}{\beta} = \frac{\gamma^* N^*}{1 - T} \Rightarrow \beta = \frac{1 - T}{1 - (\gamma^* N^* + T)},$$

where $\gamma^* N^*/(1 - T)$ is the share of income spent by the upper class in goods produced with the IRS technology relative to their income share in the economy.

in (19) to get:

$$\begin{aligned}\int_{t^*_{\widetilde{q}}}^{t} X_v^{\widetilde{q}} dv &= N^* \int_{t^*_{\widetilde{q}}}^{t} \exp(\beta g(v - t^*_{\widetilde{q}}))dv \\ &= \lambda N^* \frac{1}{\beta g} \exp(g\beta(v - t^*_q))\Big|_{t^*_q}^{t} \\ &= N^* \frac{1}{\beta g}(\exp(\beta g(t - t^*_{\widetilde{q}}) - 1) = \frac{1}{\lambda}. \qquad (22)\end{aligned}$$

We now make use of (21) and (22) to obtain:

$$g = \frac{\lambda}{\beta}(\widetilde{N}_t - N^*). \qquad (23)$$

An increase in $\widetilde{N}_t - N^*$, i.e. of the number of mass consumers who purchase goods from industries in the range $(\widetilde{q}_t, q^*_t)$ is associated, ceteris paribus, with an increase in the steady-state rate of growth. Now, using $(\beta-1)/\beta = \gamma^* N^*/(1-T)$ in (17)[13] and the solution for g obtained in (23), we get in the Pareto case the following system[14]:

$$\begin{cases} g = \beta^{-1}\lambda \widetilde{E} L(M/\alpha)(T - \widetilde{T}) \\ g = \beta^{-1}\lambda(\widetilde{N} - N^*) \end{cases}. \qquad (24)$$

Given (10), the ratio of these two equations gives:

$$\frac{1}{\widetilde{E}} = \frac{T - \widetilde{T}}{\widetilde{N}/L - N^*/L}\frac{M}{\alpha} \Leftrightarrow A_t = \widetilde{E}\frac{T - \widetilde{T}}{\widetilde{N} - N^*} y_t L. \qquad (25)$$

In Figure 2, we depict one possible solution of (24). The right quadrant of Figure 2 is a graphical representation of the cumulative income distribution $G(\gamma)$, namely the Lorenz curve, where the 45° line represents perfect equality. It shows for the bottom x% of households, what percentage y% of the total income they have. It thus describes the level of inequality in the economy. On the x-axis, one finds the cumulative share of households from lowest to highest incomes and the cumulative share of aggregate income is plotted on the y-axis.

[13] Consider (17) and use $(\beta - 1)/\beta = \gamma^* N^*/(1 - T)$, we get

$$\begin{aligned} g_t &= \lambda \widetilde{E} L \frac{M}{\alpha}\left[\frac{\beta-1}{\beta}(1-T) + T - \left(\frac{\beta-1}{\beta}\left(1-\widetilde{T}\right) + \widetilde{T}_t\right)\right] \\ g_t &= \lambda \widetilde{E} L \frac{M}{\alpha}\left[\left(1 - \frac{\beta-1}{\beta}\right)T - \left(1 - \frac{\beta-1}{\beta}\right)\widetilde{T}_t\right] \\ g_t &= \lambda \widetilde{E} L \frac{M}{\alpha}\left[\frac{1}{\beta}T - \frac{1}{\beta}\widetilde{T}_t\right]. \end{aligned}$$

[14] As g is constant in the steady state, it is also true for $\widetilde{N}$ and $\widetilde{T}$. Henceforth, we express all relations without an indication of time.

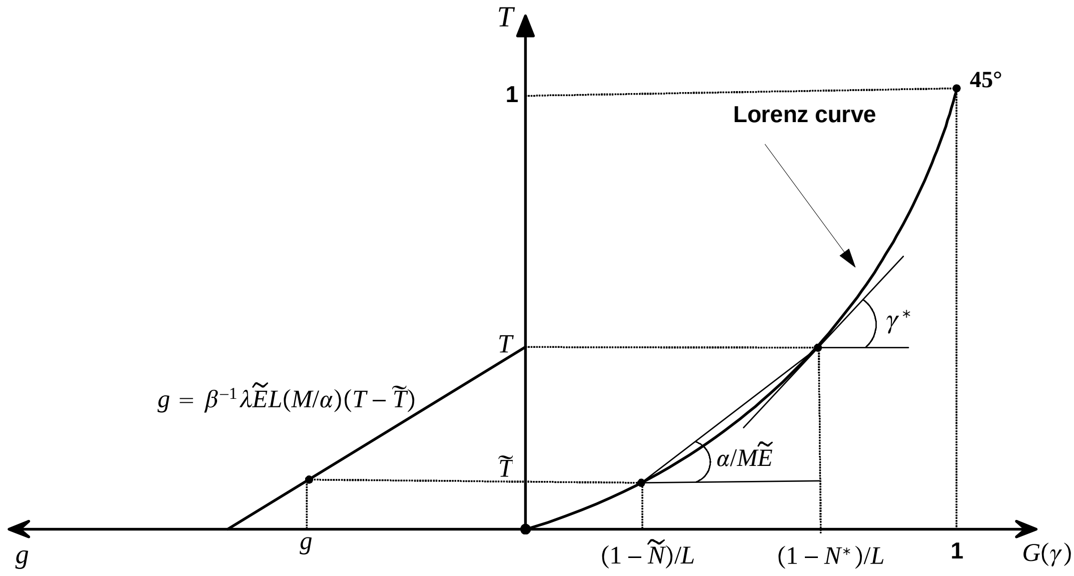


Figure 2: Inequality and the rate of growth in the steady state. Left quadrant; The rate of growth in the steady-state as provided in the first equation in system (24). Right quadrant: Lorenz curve for a Pareto distribution with a particular shape parameter β.

We then use (25) to find both $\widetilde{T}$ and $\widetilde{N}$. Recall that $\widetilde{N} - N^*$ represent the size of the middle class whose demand is responsible for the experience accumulated in the sectors $(\widetilde{q}_t, q_t^*)$, and thereby growth in equilibrium. The left quadrant is the depiction of the first equation of (24) where the long-run rate of growth is an increasing function of both the multiplier M and the income share held by households whose type lies in the range $(\widetilde{\gamma}, \gamma^*)$, i.e. $T - \widetilde{T}$.

Permanent changes in the level of inequality (β) may be represented by shifts of the Lorenz curve which will have effects on the steady state rate of growth. Consider the following example: two economies, one with perfect equality, i.e. $\beta \to \infty$, and the other characterized with complete inequality, i.e. $\beta \to 1$. In these two extreme cases, the Lorenz curves follow the 45°-line, respectively, the x-axis in the interval $(0, 1)$ and then the y-axis in 1. In the perfect equality case, note that $\widetilde{\gamma} = \gamma^*$ which implies $\widetilde{T} = T$ (or, equivalently, $\widetilde{N} = N^*$). Hence, the long run growth rate equals zero. The absence of a middle class implies that there is no industry in which learning takes place in the steady state. Thus, there is no productivity gains in the long run. Put differently, there is no transition for any industry from the CRS to the IRS technology which is the only technology where learning occurs. In the case of complete inequality, the lack of middle class obviously also leads to a zero-growth scenario in the steady state. Eventually, the difference lies in the steady-state level of development between the two economies. The most egalitarian economy underwent transitory growth. The flying geese model took place only for some time. A scenario with a take-off transition period is therefore consistent with a no sustained growth stage. In contrast, in the unequal economy, the demand is not strong enough for the flying geese model to take off with its own wings. In fact, it is very likely that no growth ever occurred at all. The existence of a middle class is thus necessary to obtain sustained growth. It is a prerequisite to a permanent structural change in which experience is accumulated at a rate which is high enough for productivity gains to allow new industries to adopt the IRS technology in the long run.

We now need to study conditions in terms of inequality and value of the various parameters for which there will be positive long-run economic growth. This is the topic of the next section.

4.2 Learning, inequality and sustained growth in the long run

Conditions on both the income distribution and parameters of the production functions are crucial which will ensure that there is sustained growth in the long run. This requires that certain conditions be met.

In the Pareto case and after some algebra[15], our variable of interest $\widetilde{T}$ in the steady state is given by:

$$\widetilde{T} = \int_{\underline{\gamma}}^{\widetilde{\gamma}} \gamma g(\gamma) L d\gamma = 1 - \left(\frac{\widetilde{N}}{L}\right)^{\frac{\beta-1}{\beta}}.$$

Let us now use the second equation in system (24) to rewrite $\widetilde{N}$ as $\min(N^* + g\beta/\lambda, L)$, and substitute it into the first equation in system (24). We get:

$$g = \frac{1}{\beta}\left(\left(\frac{\min(N^* + \frac{g\beta}{\lambda}, L)}{L}\right)^{(\beta-1)/\beta} - \left(\frac{N^*}{L}\right)^{(\beta-1)/\beta}\right)\frac{\lambda\widetilde{E}L}{1+(\alpha-1)\left(\frac{N^*}{L}\right)^{(\beta-1)/\beta}}. \tag{26}$$

$$\Rightarrow F(g) = g.$$

The issue consists essentially in characterizing the conditions under which there exists a unique fixed point $g > 0$ such that $F(g) = g$. Note first that $F(0) = 0$. Secondly, let us take advantage of the following changes in variables: $n^* = N^*/L$ and $b = (\beta - 1)/\beta$. (Keep in mind that with the Pareto distribution, $(\beta - 1)/\beta$ is equal to $\gamma^* N^*/(1 - T)$; the higher the ratio, the more the society can be regarded as egalitarian.) We thus rewrite $F(g)$ as

$$F(g) = (1-b)\left(\left(\min\left(\frac{g}{\lambda(1-b)L} + n^*, 1\right)\right)^b - (n^*)^b\right)\frac{\widetilde{E}\lambda L}{1+(\alpha-1)(n^*)^b},$$

[15] Note that:

$$\widetilde{T} = \int_{\underline{\gamma}}^{\widetilde{\gamma}} \gamma g(\gamma) L d\gamma, \text{ with } \underline{\gamma} = \frac{\beta-1}{\beta L} \text{ and } g(\gamma) = \beta\left(\frac{\beta-1}{\beta L}\right)^{\beta}\gamma^{-\beta-1}.$$

This implies

$$\begin{aligned}
\widetilde{T} &= \frac{\beta L}{1-\beta}\left(\frac{\beta-1}{\beta L}\right)^{\beta}\gamma^{1-\beta}\Big|_{\underline{\gamma}}^{\widetilde{\gamma}} \\
&= \frac{\beta L}{\beta-1}\left(\frac{\beta-1}{\beta L}\right)^{\beta}\underline{\gamma}^{1-\beta} - \frac{\beta L}{\beta-1}\left(\frac{\beta-1}{\beta L}\right)^{\beta}\widetilde{\gamma}^{1-\beta} \\
&= \left(\frac{\beta-1}{\beta L\underline{\gamma}}\right)^{\beta-1} - \left(\frac{\beta-1}{\beta L\widetilde{\gamma}}\right)^{\beta-1} \\
&\Rightarrow \widetilde{T} = 1 - \left(\frac{\beta-1}{\beta L\widetilde{\gamma}}\right)^{\beta-1} = 1 - \left(\frac{\widetilde{N}}{L}\right)^{\frac{\beta-1}{\beta}}, \text{ with } \widetilde{N} = (1 - G(\widetilde{\gamma}))L.
\end{aligned}$$

where $\partial F(g)/\partial g > 0$ and $\partial^2 F(g)/\partial g^2 < 0$, i.e. $F(g)$ is strictly concave. Hence, there exists a unique fixed point $g > 0$ as long as $\left.\partial F(g)/\partial g\right|_{g=0} > 1$. Specifically, we have

$$\frac{\partial F(g)}{\partial g} = (1-b)\frac{b}{\lambda(1-b)L}\left(\frac{g}{\lambda(1-b)L}+n^*\right)^{b-1}\frac{\widetilde{E}\lambda L}{1+(\alpha-1)(n^*)^b} > 1.$$

Evaluating $\partial F(g)/\partial g$ at $g = 0$ yields

$$\left.\frac{\partial F(g)}{\partial g}\right|_{g=0} = b(n^*)^{b-1}\frac{\widetilde{E}}{1+(\alpha-1)(n^*)^b} > 1 \Leftrightarrow \widetilde{E}b(n^*)^{b-1}-1-(\alpha-1)(n^*)^b > 0. \tag{27}$$

Let us now use the break-even condition (7), i.e. $1/n^* = (\alpha-1)L/F$, and define the function:

$$\Psi(b) = \ln\left[\left[\frac{(\alpha-1)L}{F}\right]^b + \alpha - 1\right] - \ln\left[\frac{(\alpha-1)L}{F}\right] - \ln b.$$

Put differently, the above condition (27) yields

$$\ln\widetilde{E} > \Psi(b). \tag{28}$$

The left side of (28) is just the learning upper bound whereas the right side is a function depending on $b = \gamma^* N^*/(1-T)$, i.e., the share of income spent by the upper class in goods produced with the IRS technology relative to their income share in the economy, the population size (L) and parameters characterizing both CRS and IRS production technologies, α and F. Given this last formulation, we now need to study $\Psi(b)$ in order to identify the necessary conditions for positive growth in the long run. To sum up, $\Psi(b)$ is a positive, convex function of b, tending toward $-\infty$ as $b \to 0$ and toward the value $ln\widetilde{E}^{**}$, where $\widetilde{E}^{**} = 1+F/L$, as $b \to 1$ (see Figure 3 below and Appendix 2 for details). As shown in Figure 3, we can distinguish between three scenarios. As long as $(\alpha-1)L/F > \ln\widetilde{E}^{**}$ holds, the following three cases may arise:

Case #1:

$$\widetilde{E} \leq \widetilde{E}^* \Rightarrow g = 0, \forall\ 1 < \beta < \infty, \text{ i.e. } \forall\ 0 < b < 1.$$

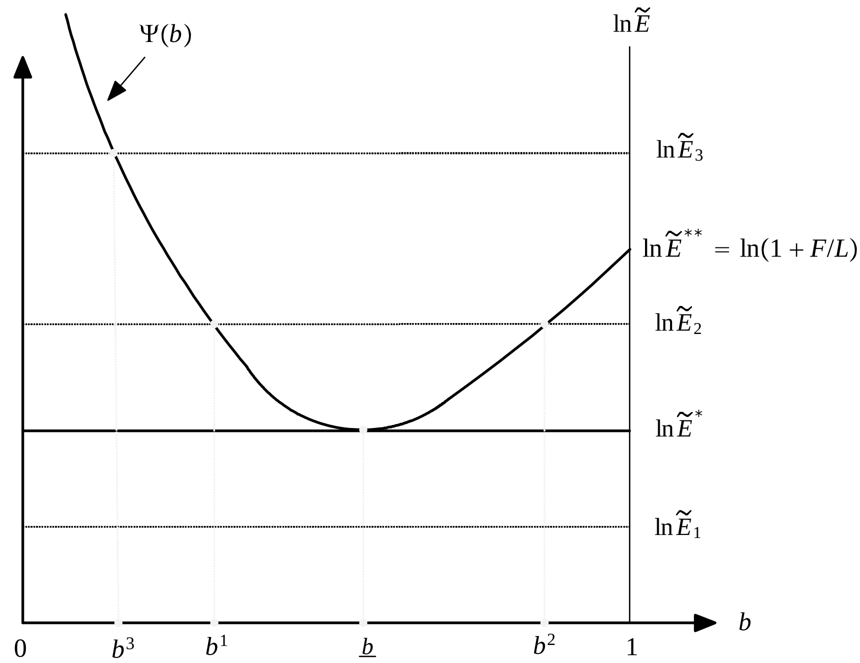
Case #2:

$$\begin{aligned}
\widetilde{E}^* &< \widetilde{E} \leq \widetilde{E}^{**}, \exists\ 1 < \beta^1 < \beta^2 < \infty, \text{ i.e. } 0 < b^1 < b^2 < 1 \\
&\quad \text{such that} \\
\text{a) if } \beta &\leq \beta^1 \Leftrightarrow b \leq b^1 \Rightarrow g = 0; \\
\text{b) if } \beta^1 &< \beta < \beta^2 \Leftrightarrow b^1 < b < b^2 \Rightarrow g > 0; \\
\text{c) if } \beta &\geq \beta^2 \Leftrightarrow b \geq b^2 \Rightarrow g = 0.
\end{aligned}$$

Case #3:

$$\begin{aligned} \widetilde{E} &> \widetilde{E}^{**}, \exists\, 1 < \beta^3 < \infty, \text{ i.e. } 0 < b^3 < 1 \\ & \text{such that} \\ \text{a) if } \beta &\leq \beta^3 \Leftrightarrow b \leq b^3 \Rightarrow g = 0; \\ \text{b) if } \beta &> \beta^3 \Leftrightarrow b > b^3 \Rightarrow g > 0. \end{aligned}$$

Otherwise, if $(\alpha - 1) L/F \leq \ln \widetilde{E}^{**}$, there is no value of b and $\widetilde{E}$ such that case #2 will occur and only case #1, i.e. $g = 0$, and case #3 may arise. As summarized in Figure 3, in Case #1, whatever the degree of equality, the potential for learning relatively to the size of the population (L), fixed costs (F), and to the marginal cost in competitive industries (α), is too weak to yield positive growth in the long run. In Case #3 instead, as soon as inequality is not too high, the learning potential in those industries that implement the IRS technology is substantial enough to generate a positive steady state rate of growth. Case #2 is more sensitive since inequality must neither be too low nor too high so that there is sustained growth.



Learning, inequality and positive growth in the long run. (See Appendix 2 for details.)

5 Mass consumption, output multiplier, and sustainable economic development

Let us start with an analysis of Fred Pearce[16], freelance journalist in England and entitled, "Consumption dwarfs population as main environmental threat":

> "Take carbon dioxide emissions - a measure of our impact on climate, [...] Stephen Pacala, director of the Princeton Environment Institute, calculates the the world's richest half-billion people -that's about 7 percent of the global population- are responsible for 50 percent of the world's carbon dioxide emissions. Meanwhile the poorest 50 percent are responsible for 7 percent of emissions... For a wider perspective of humanity's effects on the planet's life support systems, the best available measure is the "ecological footprint", which estimates the area of land required to provide each of us with food clothing, and other resources, as well as to soak up our pollution... They show that sustaining the lifestyle of the average American takes 9.5 hectares, ... and the Japanese, 4.9. The world average is 2.7 hectares. China is still below that figure at 2.1, while India and most of Africa are at or below 1.0... The carbon emissions of one American today are equivalent to those of four Chinese, 20 Indians, or 250 Ethiopians."

Following the example of Brock and Taylor (2010), it is assumed that the total emission of pollutants at time t by all industries is given by:

$$\epsilon_t = e(A_t)\int_0^{\infty} X_t^q dq = e(A_t)y_t L,$$

with $e(A_t)$, the emission of pollutants per unit of output, and $e_{A_t}(A_t) < 0$.

The pace of change in the stock of pollution (P_t) is described by:

$$\dot{P}_t = \epsilon_t - \theta P_t,$$

and θ a parameter of regeneration of the environment which reflects a mechanism opposite to that of depreciation). On the one hand, as the economic activity increases, we observe a worsening of the quality of the environment. On the other hand, the higher the stock of knowledge, the more we are able to save the quality of the environment per unit of goods produced.

Furthermore, we assume that $\overline{P}$ is an upper limit that pollution shall not exceed without producing an environmental disaster that would be irreversible. Therefore, we must have:

$$0 \leq P_t \leq \overline{P}, \forall t.$$

In the steady state, we have:

$$\dot{P}_t = 0 \Rightarrow P_t^* = \frac{\epsilon_t}{\theta} \text{ et } \frac{\dot{y}_t}{y_t} = -e_{A_t}(A_t)\frac{A_t}{e(A_t)}\frac{\dot{A}_t}{A_t},$$

[16] http://e360.yale.edu/content/feature.msp?id=2140

and (see above),

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} \Rightarrow -\frac{e_{A_t}(A_t)A_t}{e(A_t)} = -1 \Rightarrow e(A_t) = \frac{\xi}{A_t}, \text{ avec } \xi > 0.$$

Thus, to the question, is a sustainable environment compatible with economic growth in a society of mass consumption? The answer is yes, but the steady state is contingent on neutralizing the effect of the increased production ("scale effect") created by mass consumerism on the quality of the environment, by technical progress ("technical effect ") source of more efficient and cleaner technologies per unit of output. Improving the environment occurs here as a by-product of technological progress.

Moreover, if we want to avoid an environmental catastrophe, the following condition must be met:

$$P_t^* = \frac{e(A_t)y_tL}{\theta} = \frac{\xi}{\theta}\frac{y_tL}{A_t} = \frac{\xi}{\theta}\frac{M}{\alpha}L \leq \overline{P}.$$

In other words, pollution is a nondecreasing function of the population size and of the multiplier. Despite the positive impact of mass consumption on the rate of long-term growth, too many mass consumers may produce powerful enough demand spillover effects so that the economy may experience an ecological disaster while being on a balanced growth path, i.e. such that $\dot{P}_t = 0$.

6 Conclusion

[To be written]

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8 Appendices

8.1 Appendix 1: Price equilibrium

Proof. On the one hand, at time t, a monopolist entering the market for a particular good q cannot set a price higher than the competitive price without giving way to a competitive fringe of firms. On the other hand, could he seriously consider to increase its profits by lowering its price unilaterally below $p_t = \alpha w_t/A_t$, i.e., while all other firms keep their price unchanged? The answer is no, as long as the marginal profit satisfies the following condition:

$$\frac{\partial \pi_t^q}{\partial \widehat{p}_t^q} = \frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\left(\widehat{p}_t^q - \frac{w_t}{A_t}\right) + \widehat{D}_t^q > 0 \Leftrightarrow -\frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\frac{\widehat{p}_t^q}{\widehat{D}_t^q}\left(\frac{\widehat{p}_t^q - w_t/A_t}{\widehat{p}_t^q}\right) < 1, \quad (29)$$

with $\widehat{p}_t^q \leq p_t$, and where $\widehat{D}_t^q$ is the effective demand for good q produced at $\widehat{p}_t^q$. In other words, the price elasticity of demand multiplied by the price-cost margin should not exceed unity.

Let us define $\widehat{q} \leq q$ such that

$$V_t = \int_0^\infty \frac{1}{q} x_t^q dq, \quad (30)$$

$$\frac{1}{q}\frac{1}{\widehat{p}_t^q} = \frac{1}{\widehat{q}}\frac{1}{p_t} \Rightarrow \widehat{q} = \frac{\widehat{p}_t^q}{p_t} q.$$

Among the first category of households, customers for the variety of good q include all those which are rich enough to buy $\widehat{q}$, i.e., households of type $\gamma \geq \gamma_t^{\widehat{q}}$, with

$$\gamma_t^{\widehat{q}}\left(w_t \overline{h}_t L_t + \pi_t\right) =$$

$$\gamma_t^{\widehat{q}} = \frac{p_t \widehat{q}}{w_t \overline{h}_t L_t + \pi_t} = \frac{\widehat{p}_t^q q}{w_t \overline{h}_t L_t + \pi_t},.$$

Therefore, the effective demand for good q produced at price $\widehat{p}_t^q$, is

$$\widehat{D}_t^q = (1 - G_j(\gamma_t^{\widehat{q}}))L.$$

Let $g(\gamma)$ be the density of type-γ households and $\beta(\gamma) = g(\gamma)\gamma/(1 - G(\gamma))$. The price elasticity of demand for good q can be written as follows

$$\begin{aligned} -\frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\frac{\widehat{p}_t^q}{\widehat{D}_t^q} &= \frac{g(\gamma_t^{\widehat{q}})\widehat{p}_t^q q L}{\left(w_t \overline{h}_t L_t + \pi_t\right)(1 - G(\gamma_t^{\widehat{q}}))L} \\ &= \frac{\gamma_t^{\widehat{q}} g(\gamma_t^{\widehat{q}})}{(1 - G(\gamma_t^{\widehat{q}}))} \\ &= \beta(\gamma_t^{\widehat{q}}). \end{aligned} \quad (31)$$

First, we can show that using (29) and (31), we have:

$$-\frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\frac{\widehat{p}_t^q}{\widehat{D}_t^q}\left(\frac{\widehat{p}_t^q - w_t/A_t}{\widehat{p}_t^q}\right) = \beta\left(\gamma_t^{\widehat{q}}\right)\left(\frac{\widehat{p}_t^q - w_t/A_t}{\widehat{p}_t^q}\right) < \beta\left(\gamma_t^{\widehat{q}}\right)\left(\frac{p_t - w_t/A_t}{p_t^q}\right) = \beta\left(\gamma_t^{\widehat{q}}\right)\left(\frac{\alpha - 1}{\alpha}\right).$$

Therefore the following inequality provides a sufficient condition for ruling out price-cutting equilibria:

$$\beta\left(\gamma_t^{\widehat{q}}\right)\left(\frac{\alpha-1}{\alpha}\right)<1 \Rightarrow -\frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\frac{\widehat{p}_t^q}{\widehat{D}_t^q}\left(\frac{\widehat{p}_t^q-w_t/A_t}{\widehat{p}_t^q}\right)<1 \tag{32}$$

Indeed, as long as (29) is satisfied, when a firm with access to IRS technology in industry q aims to cut the price below $\alpha w_t/A_t$, it is not able to expand its customer base to such an extent as to compensate the loss in the rate of profit per customer, thus discouraging price-cutting. In our framework, such condition results in (32). The income distribution should not degenerate around any type-γ.

Let us define $G(\gamma)$ to be the Pareto distribution which has the following useful properties: (i) $\beta(\gamma)=\beta\ \forall\gamma$, and (ii) $\underline{\gamma}=(\beta-1)/(\beta L)$. Then, the condition becomes

$$\beta\left(\frac{\alpha-1}{\alpha}\right)<1 \Longleftrightarrow \beta<\frac{\alpha}{\alpha-1} \Longleftrightarrow \frac{\beta-1}{\beta}<\frac{1}{\alpha} \Longleftrightarrow b<\frac{1}{\alpha}. \tag{33}$$

where, in the text, we have defined $b=(\beta-1)/\beta$

On the other hand, when $\beta(\alpha-1)/\alpha>1 \Leftrightarrow b>1/\alpha$, the price equilibrium is equal to

$$-\frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\frac{\widehat{p}_t^q}{\widehat{D}_t^q}\left(\frac{\widehat{p}_t^q-w_t/A_t}{\widehat{p}_t^q}\right)=1 \Rightarrow \beta\left(\frac{\widehat{p}_t^q-w_t/A_t}{\widehat{p}_t^q}\right)=1 \Rightarrow$$

$$\widehat{p}_t^q=\frac{\beta}{\beta-1}w_t/A_t=\frac{1}{b}w_t/A_t<p_t=\alpha w_t/A_t.$$

In the case of a Pareto distribution, the Gini coefficient is equal to $1/(2\beta-1)$. Thus, the inequality $(\beta-1)/\beta>1/\alpha$ yields

$$Gini<\frac{\alpha-1}{\alpha+1}.$$

Let us consider a mark-up $(\alpha-1)/\alpha$ which is equal to 0.2 which means $\alpha=1.25$. In this particular case for α, the above inequality is such that $Gini<0.11$. As soon as $Gini<0.11$, the equilibrium price will differ from $\alpha w_t/A_t$ being equal to

$$p_t=\frac{\beta}{\beta-1}\frac{w_t}{A_t}.$$

Gini coefficients across countries reveal that the assumption $(\beta-1)/\beta<1/\alpha$ is more realistic. In this article, we therefore work with this assumption and the price in equilibrium is determined by $p_t=\alpha w_t/A_t$. ■

8.2 Appendix 2: Existence of a unique fixed point

We provide below a study of $\Psi(b)$ in the interval $(0,1)$, where

$$\Psi\left(b\right)=\ln\left[\nu^{b}+\alpha-1\right]-\ln\nu-\ln b,$$

and $\nu=(\alpha-1)L/F$.

First, note that $\beta=1\equiv b=0\Rightarrow\Psi(0)\rightarrow\infty$, whereas $\beta\rightarrow\infty\equiv b\rightarrow1\Rightarrow$

$$\lim_{b\rightarrow1}\Psi(b)=\ln\left[\frac{(\alpha-1)L}{F}+\alpha-1\right]-\ln\frac{(\alpha-1)L}{F}=\ln\left[1+\frac{F}{L}\right]=\ln\widetilde{E}^{**}>0.$$

Secondly, $\Psi\left(b\right)$ is convex:

$$\frac{\partial\Psi\left(b\right)}{\partial b}=\frac{\nu^{b}\ln\nu}{\nu^{b}+\alpha-1}-\frac{1}{b}\Rightarrow$$

$$\begin{aligned}\frac{\partial^{2}\Psi\left(b\right)}{\partial b^{2}} &= \frac{\left(\nu^{b}\ln\nu\right)^{2}+\left(\alpha-1\right)\nu^{b}\left(\ln\nu\right)^{2}-\left(\nu^{b}\ln\nu\right)^{2}}{\left(\nu^{b}+\alpha-1\right)^{2}}+\frac{1}{b^{2}}\\ &= \frac{\left(\alpha-1\right)\nu^{b}\left(\ln\nu\right)^{2}}{\left(\nu^{b}+\alpha-1\right)^{2}}+\frac{1}{b^{2}}>0.\end{aligned}$$

Thirdly, $\Psi\left(b\right)$ reaches a minimum value in $\underline{b}$ which is the solution to the following equation:

$$\left.\frac{\partial\Psi\left(b\right)}{\partial b}\right|_{b=\underline{b}}=0\Leftrightarrow\frac{\underline{b}\nu^{\underline{b}}\ln\nu^{\underline{b}}}{\nu^{\underline{b}}+\alpha-1}=1.$$

Finally, note that $\underline{b}$ may not necessarily lie in the interval $(0,1)$. We now analyze the value of $\partial\Psi\left(b\right)/\partial b|_{b=\underline{b}}$ inside the unit interval. We have

$$\begin{aligned}\left.\frac{\partial\Psi\left(b\right)}{\partial b}\right|_{b\rightarrow0} &= \alpha^{-1}\ln\nu-\lim_{b\rightarrow0}\frac{1}{b}\rightarrow-\infty\text{ and}\\ \left.\frac{\partial\Psi\left(b\right)}{\partial b}\right|_{b\rightarrow1} &= \frac{\nu\ln\nu}{\nu+\alpha-1}-1\gtreqless0\Leftrightarrow\nu\ln\nu-\nu-(\alpha-1)\gtreqless0.\end{aligned}$$

Replacing ν by $(\alpha-1)L/F$ eventually leads to the following important property between the population size (L), and parameters characterizing both CRS and IRS production technologies, α and F:

$$\left.\frac{\partial\Psi\left(b\right)}{\partial b}\right|_{b\rightarrow1}\gtreqless0\Leftrightarrow\ln\left(\frac{(\alpha-1)L}{F}\right)\gtreqless1+\frac{F}{L}\Leftrightarrow\frac{(\alpha-1)L}{F}\gtreqless\ln\widetilde{E}^{**}.$$